

Static spherically symmetric thin shell wormhole colliding with a spherical thin shell

Xiaobao Wang* and Sijie Gao[†]

Department of Physics, Beijing Normal University,
Beijing 100875, China

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Abstract

We consider a static spherically symmetric thin shell wormhole that collides with another thin shell consisting of ordinary matter. By employing the geometrical constraint, which leads to the conservation of energy and momentum, we show that the state after the collision can be solved from the initial data. In the low speed approximation, the solutions are rather simple. The shell may either bounce back or pass through the wormhole. In either case, the wormhole shrinks right after the collision. In the “bouncing” case, a surprising result is that the radial speeds before and after the collision satisfy an addition law, which is independent of other parameters of the wormhole and the shell. Once the shell passes through the wormhole, we find that the shell always expands. However, the expansion rate is the same as its collapsing rate right before the collision. Finally, we find out the solution for the shell moving together with the wormhole. This work sheds light on the interaction between wormholes and matter.

1 Introduction

The thin shell model[1] is an idealization of the real matter distribution and has given many interesting solutions in general relativity and alternative gravity theories. Using the “cut and paste” technique, Visser[2] proposed a simple method to construct thin shell wormholes. Linear stability of thin shell wormholes was studied later [3]-[6]. It is well known that a wormhole must contain exotic matter, which violates some energy conditions.

Usually, a wormhole is treated as a fixed background. It is important and interesting to know how a wormhole interacts with matter. For instance, how will the wormhole change when a self-gravitating object falls into it? This is a

*Email: xiaobao@mail.bnu.edu.cn

[†]Corresponding author. Email: sijie@bnu.edu.cn

difficult issue because it involves backreaction. In this paper, we use a thin shell consisting of ordinary matter as the source of perturbation. We study the physical process when an initially static wormhole collides with the ordinary shell. Investigating this process may help us understand the stability and traversability of wormholes. Although the thin shell is a simplified object, studying the problem of collision is still not easy. Notice that Langlois, Maeda and Wands [7] derived the conservation laws in the collision of thin shells. This is a consequence of the continuity of the spacetime metric. By employing the LMW mechanism as well as the Israel junction condition, we show that the problem can be solved at low speed limits. We first consider two interesting scenarios of collision: The shell bounces back or the shell passes through the wormhole. The two scenarios obey different equations of motion. For the “bouncing” case, we found that the radial speeds of the wormhole and the shell after the collision are both proportional to the original speed of the shell. Most interestingly, the three speeds satisfy a simple addition law. For the “passing” case, our solution is consistent with that in [8] which was obtained by a different approach. We also discuss the scenario that the shell and the wormhole stick together after the collision.

2 General properties for shell collisions

In this section, we review the LMW mechanism, which shows that the continuity of spacetime implies the conservation laws.

Consider a spherical shell Σ moving in a spherical spacetime. The coordinates on the two sides of the shell are labeled by (t_1, r_1) and (t_2, r_2) , where we have dropped the (θ, ϕ) coordinates for simplicity (see Fig. 1).

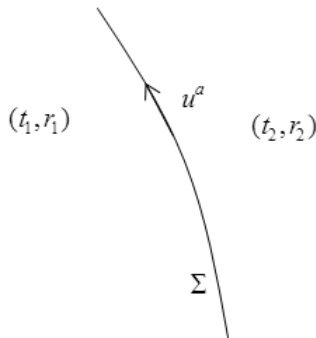


Figure 1: A spherical shell Σ moving with four-velocity u^a .

The metrics on both sides are of the form

$$ds_i^2 = -f_i(r)dt^2 + f_i^{-1}(r)dr^2 + r^2d\Omega^2 \quad (1)$$

where $i = 1, 2$ and in the Schwarzschild case

$$f_i(r) = 1 - \frac{2M_i}{r} \quad (2)$$

Let $[K_{ab}]$ be the jump of the extrinsic curvature across the shell. The evolution of the shell follows the junction condition [1]

$$[K_{ab}] = -8\pi \left(S_{ab} - \frac{1}{2} S h_{ab} \right), \quad (3)$$

where h_{ab} is the induced metric on Σ and S_{ab} is the energy-momentum tensor of the shell. In the spherical case, we have

$$\epsilon_2 \sqrt{f_2 + \dot{r}^2} - \epsilon_1 \sqrt{f_1 + \dot{r}^2} = 4\pi\sigma r, \quad (4)$$

where $\epsilon_i = -1$ corresponds to r increasing from left to right in Fig. 1 and $\epsilon_i = +1$ otherwise. For an ordinary shell, $\epsilon_1 = \epsilon_2 = -1$, while for a wormhole with exotic matter, $\epsilon_1 = 1$ and $\epsilon_2 = -1$.

Note that $r_1 = r_2$ by continuity, but t is discontinuous across the shell. We may write the four-velocity of the shell as

$$u^a = \dot{t}_i \left(\frac{\partial}{\partial t_i} \right)^a + \dot{r} \left(\frac{\partial}{\partial r_i} \right)^a \quad (5)$$

Note that we have used \dot{r} instead of \dot{r}_i because $r_1 = r_2$. The normalization condition $g_{ab}u^a u^b = -1$ yields

$$\dot{t}_i = \pm \sqrt{\frac{f_i + \dot{r}^2}{f_i^2}} \quad (6)$$

The normal vector of Σ is of the form

$$n_i^a = \frac{\dot{r}}{f_i(r)} \left(\frac{\partial}{\partial t_i} \right)^a + \sqrt{\dot{r}^2 + f_i} \left(\frac{\partial}{\partial r_i} \right)^a \quad (7)$$

Now we have three orthogonal and normal tetrads related by the following Lorentz transformation [7]

$$\begin{pmatrix} u^a \\ n^a \end{pmatrix} = \Lambda(\alpha_i) \begin{pmatrix} \sqrt{\frac{1}{f_i}} \left(\frac{\partial}{\partial t_i} \right)^a \\ \sqrt{f_i} \left(\frac{\partial}{\partial r_i} \right)^a \end{pmatrix} \quad (8)$$

where

$$\Lambda(\alpha) = \begin{pmatrix} \cosh(\alpha) & \sinh(\alpha) \\ \sinh(\alpha) & \cosh(\alpha) \end{pmatrix} \quad (9)$$

and

$$\alpha_i = \sinh^{-1} \frac{\epsilon_i \dot{r}}{\sqrt{f_i}} \quad (10)$$

Therefore,

$$\begin{pmatrix} \sqrt{\frac{1}{f_2}} \left(\frac{\partial}{\partial t_2} \right)^a \\ \sqrt{f_2} \left(\frac{\partial}{\partial r_2} \right)^a \end{pmatrix} = \Lambda(\alpha_1 - \alpha_2) \begin{pmatrix} \sqrt{\frac{1}{f_1}} \left(\frac{\partial}{\partial t_1} \right)^a \\ \sqrt{f_1} \left(\frac{\partial}{\partial r_1} \right)^a \end{pmatrix} \quad (11)$$

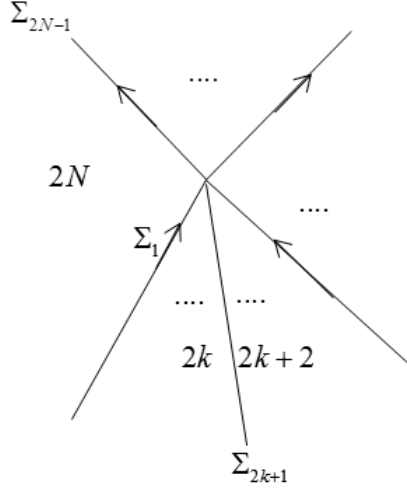


Figure 2: M shells colliding at a moment and $N - M$ shells appearing after the collision.

Now consider M shells colliding simultaneously. After the collision, $N - M$ shells appear. So there are N shells in total at the spacetime point of collision (see Fig. 2). We label each shell by an odd number and the region in between by an even number.

Define the angles on each side of the shell by

$$\sinh \alpha_{2k-1|2k} = \frac{\epsilon_{2k} \dot{r}_{2k-1}}{\sqrt{f_{2k}}} \quad (12)$$

Now perform the Lorentz transformation (11) to each shell in Fig. 2 associated with α_i near the collision point. Then after completing this process, we end up with the consistent relation

$$\prod_{k=1}^N \Lambda(\alpha_{2k-1|2k} - \alpha_{2k-1|2k-2}) = 1 \quad (13)$$

which is equivalent to

$$\sum_{i=1}^{2N} \alpha_{i|i+1} = 0 \quad (14)$$

where we have defined $\alpha_{2k|j} \equiv -\alpha_{j|2k}$. This is an important result of [7]. It is also called the geometrical constraint, which reflects the continuity of the metric at the collision point. One can show that Eq. (14), together with the junction conditions, indicates the conservation of energy and momentum.

3 Collision of a static wormhole with a shell

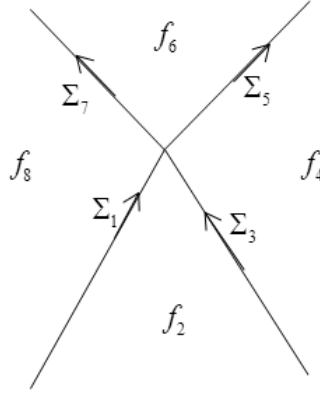


Figure 3: A wormhole Σ_1 colliding with a shell Σ_3 . The shell remains on the “right” side of the wormhole after the collision. So Σ_5 represents the shell and Σ_7 represents the wormhole.

Suppose that the spherical thin shell wormhole is originally at rest (Σ_1 in Fig. 3). Then another thin shell Σ_3 with ordinary matter collapses and hits the wormhole. After the collision, a new spacetime region with function f_6 in Fig. 3 emerges. This is the case that $M = 2$ and $N = 4$ in Fig. 2. Given initial values, we show that the junction conditions and the consistency condition (14) are just enough to determine the state after the collision. We are interested in two scenarios after the collision: The shell either remains on the same side of the wormhole or passes through the wormhole. It is reasonable to assume that the proper masses of the wormhole and the shell remain unchanged just after the collision. The two cases are determined by different sets of equations, so we shall solve them one by one. Another scenario is that the shell and the wormhole move together after the collision. In this case, the speed and the mass of the new wormhole can both be solved.

3.1 Case 1: The bouncing solution

In this case, the shell remains on the same side of the wormhole after the collision, as depicted in Fig. 3. According to Eq. (12), the Lorentz angles are given

by

$$\sinh \alpha_{18} = 0, \quad \sinh \alpha_{12} = 0 \quad (15)$$

$$\sinh \alpha_{32} = -\frac{\dot{r}_3}{\sqrt{f_2}}, \quad \sinh \alpha_{34} = -\frac{\dot{r}_3}{\sqrt{f_4}} \quad (16)$$

$$\sinh \alpha_{54} = -\frac{\dot{r}_5}{\sqrt{f_4}}, \quad \sinh \alpha_{56} = -\frac{\dot{r}_5}{\sqrt{f_6}} \quad (17)$$

$$\sinh \alpha_{76} = -\frac{\dot{r}_7}{\sqrt{f_6}}, \quad \sinh \alpha_{78} = \frac{\dot{r}_7}{\sqrt{f_8}} \quad (18)$$

The consistency condition is given by

$$\alpha_{18} - \alpha_{12} + \alpha_{32} - \alpha_{34} + \alpha_{54} - \alpha_{56} + \alpha_{76} - \alpha_{78} = 0 \quad (19)$$

It is reasonable to assume that the masses of the wormhole and the shell remain unchanged, i.e.,

$$\rho_7 = \rho_1, \quad \rho_5 = \rho_3 \quad (20)$$

For simplicity, we assume that the static wormhole is symmetric. Hence,

$$\dot{r}_1 = 0 \quad (21)$$

$$f_2 = f_8 \quad (22)$$

Applying Eq. (4) to each shell, we have the following four equations:

$$-2\sqrt{f_2} = \tilde{\rho}_1 \quad (23)$$

$$\sqrt{f_2 + \dot{r}_3^2} - \sqrt{f_4 + \dot{r}_3^2} = \tilde{\rho}_3 \quad (24)$$

$$\sqrt{f_6 + \dot{r}_5^2} - \sqrt{f_4 + \dot{r}_5^2} = \tilde{\rho}_3 \quad (25)$$

$$-\sqrt{f_2 + \dot{r}_7^2} - \sqrt{f_6 + \dot{r}_7^2} = \tilde{\rho}_1 \quad (26)$$

We may choose the initial values $f_4 (< f_2)$ and $\tilde{\rho}_3$. Then ρ_1 and \dot{r}_3 can be solved from Eqs. (23) and (24). By Eqs. (25), (26), and (19), one can solve for \dot{r}_5 , \dot{r}_7 , and f_6 .

These equations are not easy to solve, even numerically. However, in the following subsections, we derive an inequality for f_6 and then find the solutions at low speed limits.

3.1.1 An inequality

Combination of Eq. (23) and Eq. (26) yields

$$2\sqrt{f_2} = \sqrt{f_2 + \dot{r}_7^2} + \sqrt{f_6 + \dot{r}_7^2} \quad (27)$$

Solving for \dot{r}_7^2 , we have

$$\begin{aligned}\dot{r}_7^2 &= \frac{1}{16f_2}(f_6^2 - 10f_2f_6 + 9f_2^2) \\ &= \frac{f_2}{16}(p^2 - 10p + 9)\end{aligned}\quad (28)$$

where $p = \frac{f_6}{f_2}$. Since $\dot{r}_7^2 > 0$, we must have

$$f_6 < f_2 \quad \text{or} \quad f_6 > 9f_2 \quad (29)$$

This is the range for f_6 . In the following, we consider a solution of perturbation, where f_6 is very close to f_2 . Thus, only $f_6 < f_2$ will be considered.

3.1.2 Solutions for low speeds

When treating the behavior of the shell as a perturbation, it is reasonable to assume that \dot{r}_3 is small. Consequently, \dot{r}_5 and \dot{r}_7 are also small. Then by Taylor expansion, Eqs. (23)–(26) and Eq. (19) may be approximated as

$$-2\sqrt{f_2} = \tilde{\rho}_1 \quad (30)$$

$$\sqrt{f_2} + \frac{\dot{r}_3^2}{2\sqrt{f_2}} - \sqrt{f_4} - \frac{\dot{r}_3^2}{2\sqrt{f_4}} = \tilde{\rho}_3 \quad (31)$$

$$\sqrt{f_6} + \frac{\dot{r}_5^2}{2\sqrt{f_6}} - \sqrt{f_4} - \frac{\dot{r}_5^2}{2\sqrt{f_4}} = \tilde{\rho}_3 \quad (32)$$

$$-\sqrt{f_2} - \frac{\dot{r}_7^2}{2\sqrt{f_2}} - \sqrt{f_6} - \frac{\dot{r}_7^2}{2\sqrt{f_6}} = \tilde{\rho}_1 \quad (33)$$

and

$$-\frac{\dot{r}_3}{\sqrt{f_2}} + \frac{\dot{r}_3}{\sqrt{f_4}} - \frac{\dot{r}_5}{\sqrt{f_4}} + \frac{\dot{r}_5}{\sqrt{f_6}} - \frac{\dot{r}_7}{\sqrt{f_6}} - \frac{\dot{r}_7}{\sqrt{f_2}} = 0 \quad (34)$$

These equations are still not straightforward to solve. First notice that Eqs. (30) and (33) yield

$$\sqrt{f_2} - \sqrt{f_6} = \frac{\dot{r}_7^2}{2\sqrt{f_2}} + \frac{\dot{r}_7^2}{2\sqrt{f_6}} \equiv x > 0. \quad (35)$$

So

$$\sqrt{f_6} = \sqrt{f_2} - x \quad (36)$$

Now, \dot{r}_5^2 and \dot{r}_7^2 and x are in the same order. So we can replace $\sqrt{f_6}$ in the denominator of Eqs. (32) by $\sqrt{f_2}$ and obtain

$$\sqrt{f_2} - x + \frac{\dot{r}_5^2}{2\sqrt{f_2}} - \sqrt{f_4} - \frac{\dot{r}_5^2}{2\sqrt{f_4}} = \sqrt{f_2} + \frac{\dot{r}_3^2}{2\sqrt{f_2}} - \sqrt{f_4} - \frac{\dot{r}_3^2}{2\sqrt{f_4}} \quad (37)$$

i.e.,

$$-x + \frac{\dot{r}_5^2}{2\sqrt{f_2}} - \frac{\dot{r}_5^2}{2\sqrt{f_4}} = +\frac{\dot{r}_3^2}{2\sqrt{f_2}} - \frac{\dot{r}_3^2}{2\sqrt{f_4}} \quad (38)$$

Using the same approximation, Eq. (35) becomes

$$x = \frac{\dot{r}_7^2}{\sqrt{f_2}} \quad (39)$$

Substitution into Eq. (38) yields

$$-\frac{\dot{r}_7^2}{\sqrt{f_2}} + \frac{\dot{r}_5^2}{2\sqrt{f_2}} - \frac{\dot{r}_5^2}{2\sqrt{f_4}} = +\frac{\dot{r}_3^2}{2\sqrt{f_2}} - \frac{\dot{r}_3^2}{2\sqrt{f_4}} \quad (40)$$

Similarly, Eq. (34) becomes

$$-\frac{\dot{r}_3}{\sqrt{f_2}} + \frac{\dot{r}_3}{\sqrt{f_4}} - \frac{\dot{r}_5}{\sqrt{f_4}} + \frac{\dot{r}_5}{\sqrt{f_2}} - 2\frac{\dot{r}_7}{\sqrt{f_2}} = 0 \quad (41)$$

Let

$$k = \frac{\sqrt{f_4}}{\sqrt{f_2}} < 1 \quad (42)$$

Then the solution of Eqs. (40) and (41) is given by

$$\dot{r}_5 = \frac{1-3k}{k+1} \dot{r}_3 \quad (43)$$

$$\dot{r}_7 = \frac{2-2k}{k+1} \dot{r}_3 \quad (44)$$

Since

$$-1 < \frac{1-3k}{k+1} < 1 \quad (45)$$

for $0 < k < 1$, we find

$$|\dot{r}_5| < |\dot{r}_3|. \quad (46)$$

Because $\dot{r}_3 < 0$, Eq. (44) shows

$$\dot{r}_7 < 0. \quad (47)$$

Therefore, the throat of the wormhole always decreases right after the collision.

Eq. (43) shows that $\dot{r}_5 < 0$ when $k < \frac{1}{3}$, which means both the shell and the wormhole shrink. It is easy to find

$$\dot{r}_5 - \dot{r}_7 = -\dot{r}_3 > 0 \quad (48)$$

which means

$$|\dot{r}_7| > |\dot{r}_5| \quad (49)$$

if $\dot{r}_5 < 0$. So the wormhole shrinks faster than the shell. This is an expected result. Otherwise, the shell cannot remain on the same side of the wormhole after the collision, as shown in Fig. 3.

The relation

$$\dot{r}_3 + \dot{r}_5 = \dot{r}_7 \quad (50)$$

is surprisingly simple, which means that the sum of the speeds of the shell before and after the collision is equal to the speed of the wormhole after the collision!

The above analysis can be verified by numerical calculation. We choose

$$f_2 = 0.9, \quad f_4 = 0.7, \quad \dot{r}_3 = -10^{-5}. \quad (51)$$

Then Eqs. (30)–(34) can be solved numerically:

$$\dot{r}_5 = 8.745 \times 10^{-6}, \quad \dot{r}_7 = -1.255 \times 10^{-6}, \quad \sqrt{f_6} = \sqrt{f_2} - 1.660 \times 10^{-12}. \quad (52)$$

These results yield

$$\frac{\dot{r}_3 + \dot{r}_5}{\dot{r}_7} = 1.00005 \times 10^{-12} \quad (53)$$

Hence, Eq. (50) is confirmed.

3.1.3 Another solution?

There is another obvious solution:

$$\dot{r}_5 = \dot{r}_3, \quad \dot{r}_7 = 0, \quad f_6 = f_2. \quad (54)$$

The apparent interpretation of this solution is that the wormhole remains static and the shell is still collapsing with the same speed. This set of solutions even satisfies the original equations without any approximation. However, this solution is not real in physics. If the wormhole remains static and the shell remains on the same side, the shell must bounce back with a larger radius because the throat of the wormhole has the minimum radius. But this results in $\dot{r}_5 > 0$, disagreeing with $\dot{r}_5 = \dot{r}_3$. One may think that this solution indicates that the shell passes through the wormhole. If this is the case, the equations must be modified (see the next subsection). In the new configuration, we see that Eq. (54) is no longer a solution. Therefore, in either case, Eq. (54) should be discarded.

3.2 Case II: Passing through the wormhole

Now we assume that the shell travels through the wormhole and appears on the other side. In this case, the positions of Σ_7 and Σ_5 should have exchanged their roles in Fig. 3. For the “passing through” solution, Σ_5 in Fig. 3 represents the wormhole after collision and Σ_7 represents the shell.

Then Eqs. (23)-(26) are modified as

$$\begin{aligned} -2\sqrt{f_2} &= \tilde{\rho}_1 & (55) \\ \sqrt{f_2 + \dot{r}_3^2} - \sqrt{f_4 + \dot{r}_3^2} &= \tilde{\rho}_3 & (56) \end{aligned}$$

$$-\sqrt{f_6 + \dot{r}_5^2} - \sqrt{f_4 + \dot{r}_5^2} = \tilde{\rho}_1 \quad (57)$$

$$\sqrt{f_6 + \dot{r}_7^2} - \sqrt{f_2 + \dot{r}_7^2} = \tilde{\rho}_3 \quad (58)$$

Eqs. (15)-(18) become

$$\sinh \alpha_{18} = 0, \quad \sinh \alpha_{12} = 0 \quad (59)$$

$$\sinh \alpha_{32} = -\frac{\dot{r}_3}{\sqrt{f_2}}, \quad \sinh \alpha_{34} = -\frac{\dot{r}_3}{\sqrt{f_4}} \quad (60)$$

$$\sinh \alpha_{54} = -\frac{\dot{r}_5}{\sqrt{f_4}}, \quad \sinh \alpha_{56} = \frac{\dot{r}_5}{\sqrt{f_6}} \quad (61)$$

$$\sinh \alpha_{76} = \frac{\dot{r}_7}{\sqrt{f_6}}, \quad \sinh \alpha_{78} = \frac{\dot{r}_7}{\sqrt{f_2}} \quad (62)$$

For small \dot{r}_i , we have

$$-2\sqrt{f_2} = \tilde{\rho}_1 \quad (63)$$

$$\sqrt{f_2} - \sqrt{f_4} + \frac{\dot{r}_3^2}{2} \left(\frac{1}{\sqrt{f_2}} - \frac{1}{\sqrt{f_4}} \right) = \tilde{\rho}_3 \quad (64)$$

$$-\sqrt{f_6} - \sqrt{f_4} - \frac{\dot{r}_5^2}{2} \left(\frac{1}{\sqrt{f_4}} + \frac{1}{\sqrt{f_6}} \right) = \tilde{\rho}_1 \quad (65)$$

$$\sqrt{f_6} - \sqrt{f_2} + \frac{\dot{r}_7^2}{2} \left(\frac{1}{\sqrt{f_6}} - \frac{1}{\sqrt{f_2}} \right) = \tilde{\rho}_3 \quad (66)$$

and

$$-\frac{\dot{r}_3}{\sqrt{f_2}} + \frac{\dot{r}_3}{\sqrt{f_4}} - \frac{\dot{r}_5}{\sqrt{f_4}} - \frac{\dot{r}_5}{\sqrt{f_6}} + \frac{\dot{r}_7}{\sqrt{f_6}} - \frac{\dot{r}_7}{\sqrt{f_2}} = 0 \quad (67)$$

From Eqs. (63) and (65), we have

$$2\sqrt{f_2} - \sqrt{f_4} - \frac{\dot{r}_5^2}{2} \left(\frac{1}{\sqrt{f_4}} + \frac{1}{\sqrt{f_6}} \right) = \sqrt{f_6} \quad (68)$$

Therefore, $\sqrt{f_6} - 2\sqrt{f_2} + \sqrt{f_4}$ is a small quantity too. So we may replace $\sqrt{f_6}$ in the denominator of Eq. (68) with $2\sqrt{f_2} - \sqrt{f_4}$ and obtain

$$\sqrt{f_6} = 2\sqrt{f_2} - \sqrt{f_4} - \frac{\dot{r}_5^2}{2} \left(\frac{1}{\sqrt{f_4}} + \frac{1}{2\sqrt{f_2} - \sqrt{f_4}} \right) \quad (69)$$

Substituting Eq. (69) into Eqs. (66) and (67) (still replacing $\sqrt{f_6}$ in the denominator with $2\sqrt{f_2} - \sqrt{f_4}$), we find two quadratic equations of \dot{r}_5 and \dot{r}_7 . Two sets of solutions can be obtained straightforwardly:

$$\dot{r}_5 = \frac{(k-1)^2 \dot{r}_3}{1+k} \quad (70)$$

$$\dot{r}_7 = \frac{(3-k) \dot{r}_3}{1+k} \quad (71)$$

or

$$\dot{r}_5 = (1-k) \dot{r}_3 \quad (72)$$

$$\dot{r}_7 = -\dot{r}_3 \quad (73)$$

where

$$k = \frac{\sqrt{f_4}}{\sqrt{f_2}} < 1 \quad (74)$$

We see $\dot{r}_5 < 0$ for both solutions, meaning the wormhole keeps shrinking as expected. In the first solution, $\dot{r}_7 < 0$ and $|\dot{r}_7| < |\dot{r}_5|$, meaning the wormhole shrinks faster than the shell, which is inconsistent with the passing through picture. Therefore, this solution should be discarded. In the second solution, $\dot{r}_7 > 0$, meaning the shell expands after the collision. This is reasonable because when the shell appears on the other side of the wormhole, its radius must be larger than the radius of the throat of the wormhole.

It is worth mentioning that the same issue has also been discussed in [8]. By assuming that the four velocities remain unchanged, i.e.,

$$u_5^a = u_1^a, \quad u_7^a = u_3^a \quad (75)$$

the authors obtained

$$\dot{r}_5 = \tilde{\rho}_3 \frac{\dot{r}_3}{\sqrt{f_2}} \quad (76)$$

$$\dot{r}_7 = -\dot{r}_3 \quad (77)$$

We see that Eq. (77) is exactly our solution (73), while Eq. (76) reduces to Eq. (72) at the low speed limit (see Eq. (56)). This is not a coincidence because Eq. (75) guarantees the conservation law

$$m_1 u_1^a + m_3 u_3^a = m_1 u_5^a + m_3 u_7^a \quad (78)$$

which, as we have mentioned, can be derived from the consistency condition (19). However, Eq. (75) only works for the “passing” case, not the “bouncing” case.

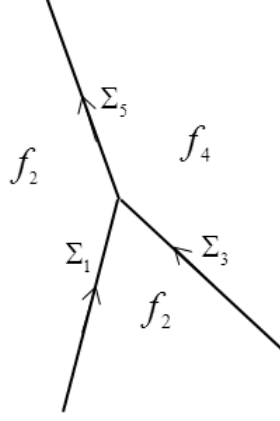


Figure 4: A new wormhole Σ_5 forms after the collision.

3.3 Case III: Moving together

Finally, we consider $M = 2$ and $N = 3$ in Fig. 2. This is the case where the shell and the wormhole stick together after the collision (see Fig. 4). We solve for the radial speed \dot{r}_5 as well as the density $\tilde{\rho}_5$.

The junction conditions for the three shells are

$$\tilde{\rho}_1 = -2\sqrt{f_2} \quad (79)$$

According to Fig. 4

$$\sqrt{f_2 + \dot{r}_3^2} - \sqrt{f_4 + \dot{r}_3^2} = \tilde{\rho}_3 \quad (80)$$

$$-\sqrt{f_2 + \dot{r}_5^2} - \sqrt{f_4 + \dot{r}_5^2} = \tilde{\rho}_5 \quad (81)$$

The Lorentz angles are given by

$$\alpha_{32} = -\frac{\dot{r}_3}{\sqrt{f_2}} \quad (82)$$

$$\alpha_{34} = -\frac{\dot{r}_3}{\sqrt{f_4}} \quad (83)$$

$$\alpha_{54} = -\frac{\dot{r}_5}{\sqrt{f_4}} \quad (84)$$

$$\alpha_{56} = \frac{\dot{r}_5}{\sqrt{f_2}} \quad (85)$$

Then in the low speed approximation, the consistency condition reads

$$\frac{\dot{r}_3}{\sqrt{f_2}} - \frac{\dot{r}_3}{\sqrt{f_4}} + \frac{\dot{r}_5}{\sqrt{f_4}} + \frac{\dot{r}_5}{\sqrt{f_2}} = 0 \quad (86)$$

We can easily find

$$\dot{r}_5 = \frac{1-k}{k+1} \dot{r}_3 \quad (87)$$

So $\dot{r}_5 < 0$, meaning that the new wormhole still shrinks. It is also easy to obtain

$$\tilde{\rho}_5 = -\sqrt{f_2 + \left(\frac{k-1}{k+1} \dot{r}_3\right)^2} - \sqrt{f_4 + \left(\frac{k-1}{k+1} \dot{r}_3\right)^2} \quad (88)$$

$$\approx -\sqrt{f_2} - \sqrt{f_4} - \frac{s^2 \dot{r}_3^2}{2\sqrt{f_2}} - \frac{s^2 \dot{r}_3^2}{2\sqrt{f_4}} \quad (89)$$

where $s = \frac{1-k}{1+k}$. So we can obtain

$$\tilde{\rho}_5 = \tilde{\rho}_1 + \tilde{\rho}_3 - \frac{(1+s^2)\dot{r}_3^2}{2\sqrt{f_2}} - \frac{(1+s^2)\dot{r}_3^2}{2\sqrt{f_4}} \quad (90)$$

This is the relation for the densities, which shows

$$\tilde{\rho}_5 < \tilde{\rho}_1 + \tilde{\rho}_3 \quad (91)$$

or

$$|\tilde{\rho}_5| > |\tilde{\rho}_1| - \tilde{\rho}_3. \quad (92)$$

The last inequality is due to the fact that $\tilde{\rho}_1 < 0$ and $\tilde{\rho}_5 < 0$.

4 Conclusions

We have investigated the collision of a static wormhole with a spherical thin shell containing ordinary matter. The junction condition of thin shells and the geometrical constraint at the collision event play crucial roles in the process. When the shell hits the wormhole at a low speed, the radial speeds of the wormhole and the shell after the collision are proportional to the initial speed. When the shell bounces back, we have found that the radial speed of the wormhole after collision is equal to the sum of the speeds of the shell before and after the collision. This result has been verified numerically. If the shell goes through the wormhole, our results are consistent with those in [8]. Our methods apply to a wide class of spherical spacetimes and can be generalized to other collisions, for instance, the collision of two moving thin shells. We have been focusing on calculating the data right after the collision. The evolutions of the shells after the collision depend on the equations of states. Our calculation provides the initial data for the evolutions.

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